**1.** An engineer at Fertilizer Company has synthesized a sensational new fertilizer made of just two interchangeable basic raw materials. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company currently has $180 to buy raw materials at a unit price of $8 and $5 per unit, respectively. When amounts *x1* and *x2* of the basic raw materials are combined, a quantity *q* of fertilizer results given by:

Part A: Formulate as a constrained nonlinear program. Clearly indicate the variables, objective function, and constraints.

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

1. Decision Variables:

: Amount of the first raw material used.

Amount of the second raw material used.

2. Objective Function: The objective is to maximize the amount q of fertilizer. So, the objective function is:

Since we want to maximize q, our nonlinear program is:

3. Constraints: We have a budget constraint based on the unit price of each raw material. The budget constraint is:

Additionally, since we cannot purchase a negative amount of raw materials:

The constrained nonlinear program is:

Decision Variables:

Objective Function:

Constraints:

A computer screen shot of a program code

Description automatically generated

**2.** A neighbor is looking to build a fenced enclosure for his chickens and wants to build using fencing he found in his local farm supply store. He can buy at most 100 feet of fencing, and the price per foot of the fencing is twice the square root of the length purchased. So he can purchase 100 feet of fencing for $20, 64 feet for $16, etc. He must also purchase fence posts to reinforce the fencing, and due to wind conditions in his yard, he must purchase more posts for east-west fencing than north-south fencing. The cost for posts in the east-west direction is $4 per foot while it is $3 per foot in the north-south direction. He has $20 to spend on fencing and $150 to spend on fence posts.

Part A: Formulate the problem as a constrained nonlinear program that will enable us to maximize the area of the fenced area, with constraints. Clearly indicate the variables, objective function, and constraints.

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

Decision Variables:

Length of the fencing in the east and west directions (feet).

Length of the fencing in the north and south directions (feet).

Objective Function: The objective is to maximize area A of the fenced area. Since the fenced area is a rectangle, the area is . Therefore:

Maximize

Constraints:

1. Total Fencing Constraint: Given the total length of the fencing he can buy is at most 100 feet:

2. Fencing Cost Constraint: The price per foot of fencing is twice the square root of the length purchased. So, the cost of x feet of fencing is , and the cost of feet of fencing is . The total fencing cost is:

3. Fence Post Cost Constraint: Given the cost for fence posts in the east and west directions is $4 per foot and it is $3 per foot in the north and south directions:

4. Non-negativity constraints (since we cannot have negative lengths of fencing):

Summarizing, the constrained nonlinear program is:

Decision Variables:

Objective Function:

Maximize

Constraints:

A screen shot of a computer program

Description automatically generated

**3.** Toy-Vey makes three types of new toys: tanks, trucks, and turtles. It takes two hours of labor to make one tank, two hours for one truck, and one hour for a turtle. The cost of manufacturing one tank is $7, 1 truck is $5 and 1 turtle is $4; a target budget of $164,000 is initially used as a guideline for the company to follow. Material requirements for the toys are shown below.



Management has rank-ordered three goals it wishes to achieve, arranged from highest to lowest priorities.

1. Minimize labor hours over 10,000 hours a week for production (40 hours for each of the 250 employees)
2. Minimize over-utilization of the weekly available supply of materials used in making the toys and place twice as much emphasis on the plastic
3. Minimize the under and over-utilization of the budget. Maximize available labor hour usage

Formulate the above decision problem as a single linear goal program. Do not solve.

\*Bonus (5 points): Solve the problem and give the number of each toy to produce as well as any violations of the goals (weights don’t have to add up to 1; use simple weights – 1, 2, 3, etc.).

Objective: Minimize the total costs associated with the production of toys, which include the costs for exceeding labor and plastic usage targets as well as any deviations (both positive and negative) from the budget.

Decision Variables:

* Number of Tanks to be produced (Integer)
* Number of Trucks to be produced (Integer)
* Number of Turtles to be produced (Integer)
* Excess labor hours used beyond the target (Continuous)
* Excess plastic used beyond the target (Continuous)
* Positive deviation from the budget (Continuous)
* Negative deviation from the budget (Continuous)

Objective Function:

Minimize:

The function aims to minimize costs related to:

* Excess labor (3 times the excess hours – weight 3)
* Excess plastic usage (4 times the excess amount to give double emphasis – weight 2 multiplied by emphasis 2)
* Positive and negative deviations from the budget (weight 1)

Constraints:

1. Labor Limit: The production of toys should not lead to an excess of labor beyond the stipulated limit.

2. Plastic Limit: The plastic used in toy production should not exceed the assigned quota.

3. Rubber Limit: The rubber utilization for toy manufacturing must be within the defined boundary.

4. Metal Limit: Metal consumption for the toys should not surpass its given threshold.

5. Budget Balance: The total cost of producing toys should align with the budget. Any deviation (overspending or underspending) from this budget should be tracked.

6. Non-negativity: All the variables are non-negative.

7. The variables are integer-valued, reflecting the fact that we cannot produce a fraction of a toy.

A computer screen shot of a program code

Description automatically generated

4. Breaking Ad is planning its advertising campaign for a customer’s new product and is going to leverage podcasts and YouTube for its advertisements. The total number of exposures per $1,000 is estimated to be 10,000 for podcasts and 7,500 for YouTube. The customer sees the campaign as successful if 750,000 people are reached and consider the campaign to be superbly successful if the exposures exceed 1 million people. The customer also wants to target its two largest age groups: 18 – 21 and 25 – 30. The total number of exposures per $1,000 for these age groups are shown below.



Management has rank ordered five goals it wishes to achieve, arranged from highest to lowest priorities.

1. Successful campaign – at least 750,000 exposures
2. Limit advertising costs to $100,000.
3. Limit podcast advertising costs to $70,000
4. Superbly successful campaign – at least 1 million exposures
5. Achieve at least 250,000 exposures for each of the two age groups. However, as the 25 – 30 age group has more buying power, double the emphasis on this age group over the 18 – 21 age group

Formulate the above decision problem as a single linear goal program.

\*Bonus (5 points): Solve the problem and give the expenditures for each media advertising campaign as well as any violations of the goals (weights don’t have to add up to 1; use simple weights – 1, 2, 3, etc.).

Decision Variables:

* Amount of money (in thousands of dollars) invested in podcast advertising.
* Amount of money (in thousands of dollars) invested in YouTube advertising.

Positive Deviation Variables:

* represent the positive deviations for goals 1, 2, 3, 4, 5a, and 5b, respectively.

Negative Deviation Variables:

* represent the negative deviations for goals 1, 2, 3, 4, 5a, and 5b, respectively.

Constraints:

* Goal 1: Successful campaign – at least 750,000 exposures
* Goal 2: Limit advertising costs to $100,000
* Goal 3: Limit podcast advertising costs to $70,000
* Goal 4: Superbly successful campaign – at least 1 million exposures
* Goal 5a: Achieve at least 250,000 exposures for the 18-21 age group
* Goal 5b: Achieve at least 250,000 exposures for the 25-30 age group with double emphasis

Objective Function:

Minimize:

These weights have been assigned to the objective function, as per the goals rank order:

* Goal 1 – 5
* Goal 2 – 4
* Goal 3 – 3
* Goal 4 – 2
* Goal 5 – 1

For goal 5b, the coefficients for the positive and negative deviations are doubled to reflect the increased emphasis on the 25-30 age group.

A computer screen shot of a program

Description automatically generated

Validating the solution:

Decision Variables:

* x = Investment in podcasts = $70,000
* y = Investment in YouTube = $40,000

Constraints:

1. Goal 1: Successful campaign – at least 750,000 exposures

The deviation would be 250,000 for this goal.

2. Goal 2: Limit advertising costs to $100,000

The positive deviation dP2 would be -10 for this goal.

3. Goal 3: Limit podcast advertising costs to $70,000

The investment in podcasts is $70,000, so dP3 is 0.

4. Goal 4: Superbly successful campaign – at least 1 million exposures

10,000(70) + 7500(40) – dN4 = 1,000,000

Using our calculations from Goal 1, the deviation dN4 is 0.

5. Goal 5a: Achieve at least 250,000 exposures for the 18-21 age group

250,000

The deviation dN5a would be 45,000 for this goal.

6. Goal 5b: Achieve at least 250,000 exposures for the 25-30 age group with double emphasis

The deviation dN5b would be 20,000 for this goal.

Summary: The solution meets all the goals, except goal 2, with deviations as calculated above. Given the objective function, we wanted these deviations to be as low as possible since they're being minimized. However, it does exceed the budget in Goal 2 by $10,000, which is represented by the deviation dP2.

5. A local farmer’s market sells, among other things, fresh apples during the harvest season. The market has $750 to purchase bushels of apples from orchard 1 at $5 per bushel, orchard 2 at $6 per bushel, or orchard 3 at $8 per bushel. However, the quality of the apples varies by orchard and the market can earn $10 per bushel from orchard 1, $11 per bushel from orchard 2, and $20 per bushel from orchard 3. Orchard 3 is selective with its sales and will only sell between 20 and 40 bushels to the market. That is, it will not sell to the farmer’s market if they order fewer than 20 bushels and will not sell more than 40 bushels to the market. Further, orchard 1 only has 50 bushels available to sell.

Part A: Formulate this as a nonlinear program by using an indicator variable to maximize the market’s profit

Part B: Solve the problem and give the solution (decision variables and objective function).

Let represent the number of bushels the market buys from orchard 1, orchard 2, and orchard 3 respectively. Also, let be an indicator variable such that:

Objective:

Maximize Profit = Revenue - Costs

Revenue:

Costs:

So, the objective function becomes:

Constraints:

1. Total budget constraint:

2. Limit on availability from orchard 1:

3. Constraint for orchard 3:

This ensures that will be between 20 and 40 if and will be 0 if .

4. Non-negativity constraints:

Nonlinear Program is:

Maximize:

Subject to:

1.

2.

3.

4.

5.

6.

A screen shot of a computer program

Description automatically generated